Power Switching in Hybrid Coherent Couplers

W. D. Deering[1] and M. I. Molina[2]

[1] *Center for Nonlinear Science, Department of Physics*
University of North Texas, Denton, TX 76203, USA

[2] *Departamento de Física, Facultad de Ciencias*
Universidad de Chile
Casilla 653, Santiago, Chile

Abstract

We report on a theoretical and numerical investigation of the switching of power in new hybrid models of nonlinear coherent couplers consisting of optical slab waveguides with various orders of nonlinearity. The first model consists of two guides with second-order instead of the usual third-order susceptibilities as typified by the Jensen coupler. This second-order system is shown to have a power self-trapping transition at a critical power greater than the third-order susceptibility coupler. Next, we consider a mixed coupler composed of a second-order guide coupled to a third-order guide and show that, although it does not display a rigorous self-trapping transition, for a particular choice of parameters it does show a fairly abrupt trapping of power at a lower power than in the third-order coupler. By coupling this mixed nonlinear pair to a third, purely linear guide, the power trapping can be brought to even lower levels and in this way a satisfactory switching profile can be achieved at less than one sixth the input power needed in the Jensen coupler.

1 Introduction

Interest in all-optical switching devices has led to the study and design of several promising configurations of nonlinear couplers which display intensity-triggered power switching. The basic nonlinear coherent coupler, introduced by Jensen [1], consists of two similar waveguides made of a material with third-order susceptibilities, embedded in a host with purely linear susceptibility. When the guides are placed parallel to each other and in close proximity over a given distance, the guide fields overlap to some extent and power can be transferred between the two. When all the power is initially launched into one of the guides, the nonlinear susceptibility can give rise to self-trapping of power in the original guide. The output power in the original guide, for a device length equal to a coupling length, can be made to switch from essentially zero percent at low power levels, to one hundred percent for input power levels exceeding a characteristic threshold. In addition to the pioneering work by Jensen, several other coupler configurations have been considered. It was found that a three-in-a-line configuration of couplers displays a more abrupt switching profile, at the expense however, of greater input power[2]. The same tendency was reported for a linear array of many couplers[3]. In an effort to improve the switching profile, we introduced in a recent work[4] the Doubly Nonlinear Trimer (DNT) coupler consisting of two nonlinear guides coupled to a third, linear guide. Such a system displays the interesting phenomenon of power self-trapping *tunability*: the critical input power level necessary for the onset of power self-trapping can be tuned to low values, by adjusting the value of the (linear) coupling between the nonlinear guides and the linear one.[4],[5] In the optimal configuration, switching was achieved at one-fourth the power needed to produce switching in the Jensen coupler. The price to pay for this improved switching is the use of larger device lengths, up to ten times that reported by Jensen[4].

In the present work, our interest is in learning if couplers having waveguides with differing types of nonlinear susceptibilities would have better switching characteristics than other standard models. We first investigate a different nonlinear coupler composed of two identical guides made of optical material lacking inversion symmetry and therefore having a nonvanishing

second-order susceptibility. We show that this new coupler array possesses a power self-trapping transition and an associated sharp power switching profile, albeit at a larger input power level than in Jensen's and in our earlier DNT coupler. Then, after examining a number of two-guide couplers of mixed compositions, with each guide having purely linear (L), and second-order (SO) or the usual third-order (TO) susceptibilities we found that for a particular choice of parameters, a coupler composed of an SO guide and a TO guide displays a relatively sharp power self-trapping profile at an input power level lower than previously reported, if power is initially launched in the SO guide. Next, as in the DNT case, the onset of self-trapping can be tuned to even lower power levels, by perturbing the two-guide coupler by adding a purely linear control guide and adjusting the strength of the interaction with this third guide. The resulting three-guide coupler, dubbed SO-TO-L, resembles the DNT configuration, with one of the third-order guides replaced by a second-order guide; it displays a reasonably sharp switching profile and, as far as we know, does so at the lowest input power reported so far.

2 A new two-guide coupler

Consider a linearly coupled system of two nonlinear guides, each having the same second-order nonlinear susceptibility. In the single mode approximation, the normalized mode amplitudes satisfy

$$i\frac{dC_1}{dz} = VC_2 - \chi|C_1|C_1 \tag{1}$$

$$i\frac{dC_2}{dz} = VC_1 - \chi|C_2|C_2, \tag{2}$$

where $\chi = Q^{(2)}\sqrt{P}$ is the product of an integral $Q^{(2)}$ containing the second-order nonlinear susceptibility[1] and the square root of the input power P. The linear coupling of the guides is determined by the coefficient V. With all the power initially launched in guide 1, the initial conditions are $C_1(0) = 1$, $C_2(0) = 0$. We will now show that Eqns.(1)-(2) predicts a *self-trapping* of power in the original guide (guide 1). First, it is convenient to rewrite (1-2) as a set of four equations for the complex quantities $\rho_{ij} \equiv C_i C_j^*$:

$$i\frac{d\rho_{11}}{dz} = -V(\rho_{12} - \rho_{21}) \tag{3}$$

$$
\begin{aligned}
i\frac{d\rho_{22}}{dz} &= V(\rho_{12}-\rho_{21}) && (4)\\
i\frac{d\rho_{12}}{dz} &= -V(\rho_{11}-\rho_{22})\\
&\quad +\chi(\sqrt{\rho_{22}}-\sqrt{\rho_{11}})\rho_{12} && (5)\\
i\frac{d\rho_{21}}{dz} &= V(\rho_{11}-\rho_{22})\\
&\quad -\chi(\sqrt{\rho_{22}}-\sqrt{\rho_{11}})\rho_{21}. && (6)
\end{aligned}
$$

We have two conserved quantities: the total power, normalized to unity: $\rho_{11}+\rho_{22}=1$ and the total "energy" $H = V(\rho_{12}+\rho_{21}) - (2/3)\chi(\rho_{11}^{3/2} + \rho_{22}^{3/2}) = -(2/3)\chi$ leaving only two independent unknowns, which precludes any chaotic dynamics for the system. Making use of these conserved quantities we find, after some tedious algebra, the following first-order equation for $\rho_{11}\equiv\rho$:

$$\frac{1}{2}(\frac{d\rho}{dz})^2 + U(\rho) = 0 \tag{7}$$

with

$$
\begin{aligned}
U(\rho) &= -2\rho(1-\rho) + \frac{1}{2}\left(\frac{2\chi}{3V}\right)^2\\
&\quad -\frac{2}{3}\left(\frac{\chi}{V}\right)^2\sqrt{1-\rho}\left(-\frac{2}{3}\rho^{3/2}(1-\rho)\right)\\
&\quad -\frac{1}{3}(\rho^3+(1-\rho)^3)\\
&\quad +\frac{2}{3}(\rho^{3/2}+(1-\rho)^{3/2}). && (8)
\end{aligned}
$$

Equation (7) describes a classical particle of unit mass, moving under the influence of an external potential $U(\rho)$, with initial condition $\rho(0)=1$. Fig.1 shows the effective potential $U(\rho)$ for several different values of χ/V. For small nonlinearity values, the effective potential is concave and conservation of energy allows complete oscillations of the "particle"; that is, power is transferred between the two guides. As nonlinearity (input power) is increased, the potential develops a local maximum whose height increases with increasing nonlinearity. The condition for self-trapping of power in the original guide translates here into the condition for the potential $U(\rho)$ to develop a double root at $\rho=\rho^*$ for some critical value of χ/V, i.e., $U(\rho^*)=0$ and

$(dU/d\rho)_{\rho^*} = 0$. Close examination of Eq.(8) and Fig.1 reveals $U(\rho)$ to be *even* around $\rho = 1/2$ and that $\rho^* = 1/2$. From that, the critical value of the nonlinearity is obtained in closed form as

$$\left(\frac{\chi}{V}\right)_c = \left(\frac{3}{\sqrt{2}}\right)\sqrt{3 + 2\sqrt{2}} \approx 5.121. \tag{9}$$

This value is greater than the critical values for Jensen's coupler ($= 4$) and for the array of three nonlinear (third-order) couplers[2] (≈ 4.5). Figure 2 shows the average transmittance of the guide, defined as

$$< P > \equiv \lim_{L\to\infty} (1/L) \int_0^L \rho(z)dz. \tag{10}$$

Clearly, we see that for $(\chi/V) < (\chi/V)_c$, power is equally distributed between the two guides. At $(\chi/V) = (\chi/V)_c$, an abrupt transition takes place and power begins to self-trap in the original guide. Onset of self-trapping is a precursor for the appearance of a sharp switching profile in the transmittance of the guide. The transmittance, defined as $|C_1(L_c)|^2$, is the quantity of basic interest for optics. The length L_c is usually chosen as the shortest length for which $|C_1(z)|^2$ is zero, or very nearly so, in the absence of nonlinearity ($\chi = 0$). In the case of the two waveguide system, $L_c = \pi/(2V)$. The abrupt increase in transmittance caused by an increment of the nonlinearity parameter (input power) can be used as a power triggered switch[1].

Figure 3 shows the transmittance characteristics of our two-guide second-order (SO) coupler, and compares it with Jensen's third-order (TO) nonlinear coupler which is also shown in the figure, along with the TO nonlinear coupler with three guides[2]. We note the SO nonlinear coupler array does not have a competitive switching profile compared to Jensen's and the three-coupler array.

3 A New Hybrid Configuration

After considering the above nonlinear coupler, having second-order susceptibility, we next examined a variety of mixed two-guide couplers in which each guide was either a purely linear one, a SO or a TO guide. The objective was to find other two-guide couplers that displayed power self-trapping for the initial condition where all the initial power is put into one guide. We found

that, in most cases there is no self-trapping transition at all but a continuous power trapping. For a given mixed two-guide coupler, the trapping profile depends in a sensitive way on the order of the nonlinear susceptibility of the guide initially receiving all power. To illustrate this point, we now describe the most interesting case we found: The SO-TO guide system, where guide 1 possesses a second-order nonlinear susceptibility integral[1] $Q_1^{(2)}$ and guide 2 possesses the usual third-order susceptibility integral[1] $Q_2^{(3)}$. The equations for the mode amplitudes are

$$
\begin{aligned}
i\frac{dC_1}{dz} &= VC_2 - \chi_1|C_1|C_1 \qquad (11)\\
i\frac{dC_2}{dz} &= VC_1 - \chi_2|C_2|^2C_2, \qquad (12)
\end{aligned}
$$

where $\chi_1 = Q_1^{(2)}\sqrt{P}$ and $\chi_2 = Q_2^{(3)}P$. When all initial input power goes into the TO guide (#2), the initial condition for the system, Eqns. (11)-(12), is $C_1(0) = 0$, $C_2(0) = 1$. A numerical investigation of $< P >$ reveals a "delayed" self-trapping transition at $\chi_1 = \chi_2 = \chi_c \approx 6.3\ V$ (Fig.4). This value is much greater than Jensen's and is, therefore, not useful for our purposes. On the other hand, when all input power is put initially into the SO guide (#1), we have the initial condition $C_1(0) = 1$, $C_2(0) = 0$. In this case, a numerical search reveals that this system does not show a self-trapping transition: the effective potential $U(\rho, \chi_1, \chi_2)$ does not develop a double root for any combination of χ_1, χ_2. However, for the special case $\chi_1 = \chi_2 \equiv \chi$, we found a relatively sharp power self-trapping profile occurring at $\chi \approx 3.0\ V$ (Fig.4); *i.e.*, a smaller power than Jensen's critical value for self-trapping. We then proceeded to "tune" the trapping profile to even lower power levels, by allowing the SO-TO coupler to interact linearly with a third (control) guide possessing only linear susceptibility. The enlarged set of equations for the mode amplitudes in this SO-TO-L coupler now reads

$$
\begin{aligned}
i\frac{dC_1}{dz} &= VC_2 + WC_3 - \chi|C_1|C_1 \qquad (13)\\
i\frac{dC_2}{dz} &= VC_1 + WC_3 - \chi|C_2|^2C_2 \qquad (14)\\
i\frac{dC_3}{dz} &= W(C_1 + C_2), \qquad (15)
\end{aligned}
$$

with initial conditions $C_1(0) = 1$, $C_2(0) = C_3(0) = 0$. It is assumed here that the guides have the same *linear* susceptibility, to minimize possible phase

mismatch effects. After examining $< P >$ as a function of χ for different W values, we found that $W \approx 1.1V$ brings the onset of self-trapping down to a power level $\chi \approx 0.4\ V$. Note that this optimal W value is the same as found for the DNT coupler[4]. Now, to evaluate the transmittance of this SO-TO-L array, we need to calculate the coupling length $L_c(W)$. This is obtained from Eqns. (13)-(15) as the position z at which $|C_1(z)|^2 \approx 0$, for $\chi = 0$. In this limit the system of equations can be solved in closed form[4] and yields for $|C_1(z)|^2$:

$$\begin{aligned} |C_1(z)|^2 \quad = \quad & A \cos\left[\left(\frac{3V - \sqrt{V^2 + 8W^2}}{2}\right) z\right] \\ & + B \cos\left[\left(\frac{3V + \sqrt{V^2 + 8W^2}}{2}\right) z\right] \\ & + C \cos[\sqrt{V^2 + 8W^2} z] + D, \end{aligned} \tag{16}$$

where

$$A = \left(\sqrt{V^2 + 8W^2} - V\right) / \left(4\sqrt{V^2 + 8W^2}\right)$$
$$B = \left(\sqrt{V^2 + 8W^2} + V\right) / \left(4\sqrt{V^2 + 8W^2}\right)$$
$$C = W^2 / \left(V^2 + 8W^2\right)$$
$$D = \left(V^2 + 4W^2\right) / \left[4\left(V^2 + 8W^2\right)\right] + 1/4.$$

For $W = 1.1\ V$, Eqn.(16) gives $L_c \approx 21/V$, the same value as for the DNT coupler. Figure 5 shows the transmittance of the SO-TO-L system as a function of input power, for the optimal linear coupling value $W = 1.1V$. For comparison we also show the transmittance for the DNT coupler. Jensen's device switches at about $\chi = 4V$ and the side-by-side three-nonlinear guide coupler of ref. 2 switches at about $\chi \sim 4.5\ V$, but because of the scale of the figure, neither of these transitions is shown. We note that the new coupler configuration SO-TO-L is capable of achieving over 99% power switching for input power levels below $\chi \sim 0.65\ V$ which is a 48% reduction in input power needed compared to the DNT device.

4 Discussion

In order for the above results to be meaningful, it must be true that χ_2 and χ_3 can be at least approximately equal for some materials. These coefficients involve the usual susceptibilities $\chi^{(j)}$ defined here to give the electric polarization P_E in the form

$$P_{Ei} = \epsilon_0 \left[\chi_{ij}^{(1)} E_j + \chi_{ijk}^{(2)} E_j E_k + \chi_{ijkm}^{(3)} E_j E_k E_m + \cdots \right].$$

To find the ratio $\chi_2/\chi_3 = Q_2/\left(Q_3\sqrt{P}\right)$, we use the definitions from ref. 1 of the integrals Q_2 and Q_3, inserting the exact expressions for mode fields and susceptibilities. Rather than going through those calculations, we make the simplifying assumptions that the $\chi^{(j)}$ are constant across each guide and that the mode field is also constant (approximately true for the TE_0 mode) across the guide; then the integrals are easily done and we get

$$\chi_2/\chi_3 \simeq \frac{\chi^{(2)}}{\chi^{(3)} \left| E \right| \sqrt{P}},$$

where P is the input power and $|E|$ is the amplitude of a slab waveguide mode field, normalized to one watt/meter. Then the ratio χ_2/χ_3 can be on the order of unity within the range of known values of the susceptibilities[6] and power in the range 0.01 - 1 kw.

As mentioned previously, the critical length L_c for the SO coupler is the same as for the Jensen coupler, but the SO device switches less abruptly and at higher power than Jensen's. The SO-TO coupler shows final-state asymmetry depending on which guide receives input power. If power enters the TO leg, a self-trapping transition occurs at more than 1.5 times the Jensen level, P_J. If the SO leg receives the power, a relatively sharp self-trapping sets in at about 25% below P_J.

A greatly lowered power switching level is shown by SO-TO-L, but its L_c is an order of magnitude larger than the Jensen L_c. Typical values for L_c are about a millimeter[7] for weakly coupled devices (*i.e.*, the separations between waveguides are large enough that coupled-mode theory can be used) and less for stronger coupling. Then L_c for SO-TO-L is on the order of a centimeter or less.

The linear interaction coefficients V and W are overlap integrals, across one waveguide, of the product of the electric mode field of that guide and

the mode field of a second guide. Therefore, V and W are functions of the separation of the waveguides and in principle, it is possible to alter one without changing the other; that is, the system can be tuned to achieve minimum power switching level, by changing the distances between the linear guide and the other two, nonlinear guides.

5 Conclusions

Our primary interest was the investigation of switching characteristics of model nonlinear couplers having mixtures of waveguides, not necessarily with the same orders of nonlinear susceptibilities. Earlier work on the DNT system suggested tunability might also be used in a hybrid coupler to decrease switching power levels. It appears possible to meet the condition $\chi_2 \cong \chi_3$, as far as known values of these quantities are concerned. Whether specific materials can be found that meet this condition and are also compatible with one another in a device, is another matter and one we have not addressed in this paper .

Switching characteristics of SO is inferior to the TO system. For SO-TO, the asymmetry of final states with respect to input guide may be the only aspect of its performance that could be of interest.

The most interesting coupler was the SO-TO-L, formed by adding a linear guide to SO-TO and tuning for minimum power by adjusting the relative positions of the guides. The transition power level drops to less than one-sixth of P_J. Although a disadvantage of this coupler is a critical length that is longer than for the Jensen coupler by an order of magnitude, that may be tolerable in some applications.

Of course, there are various other configurations involving arrays of these couplers but those were not investigated.

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FIGURE CAPTIONS

Figure 1: SO coupler system: Effective classical potential $U(\rho)$ for a particle of unit mass and "coordinate" ρ which, at $z = 0$ starts from $\rho = 1$. The condition for the onset of a power self-trapping transition is the appearance of a double root at some value of nonlinearity (input power). For our SO coupler, $(\chi/V)_c = 5.12$.

Figure 2: Space-averaged transmittance $< P >$ for the SO coupler (solid line). At $(\chi/V) = 5.12$ there is a power self-trapping transition and power begins to selftrap in the original guide. For comparison we also show $< P >$ for Jensen's coupler (dashed line).

Figure 3: Transmittance $|C_1(L_c)|^2$ versus the power parameter χ/V for the SO nonlinear coupler (solid line, $L_c = \pi/2\ V$), Jensen's coupler (dotted line, $L_c = \pi/2\ V$) and the three-in-a-line TO nonlinear coupler (dashed line, $L_c = \pi/\sqrt{2}\ V$).

Figure 4: Space-averaged transmittance for the SO-TO coupler. When all the initial input power goes to the TO guide, we have a "delayed" self-trapping transition around $\chi/V \approx 6.3$ (dashed line). If the initial power is put into the SO guide, there is no self-trapping transition, but at $(\chi/V) \approx 3.0$ power begins to self-trap in the original guide in a reasonably sharp manner (solid line).

Figure 5: Transmittance $|C_1(L_c)|^2$ versus power parameter χ/V for the SO-TO-L coupler (solid line) and the DNT coupler (dashed line). In both cases $L_c \approx 21/V$.

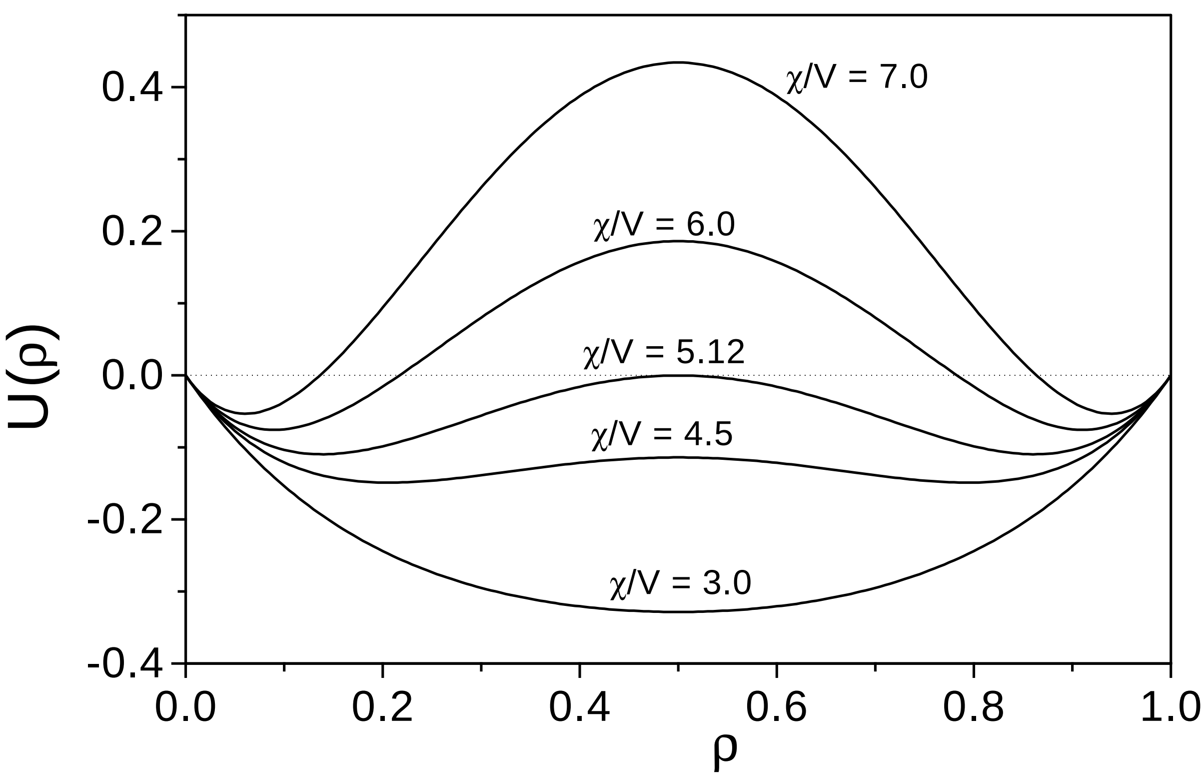


Fig.1

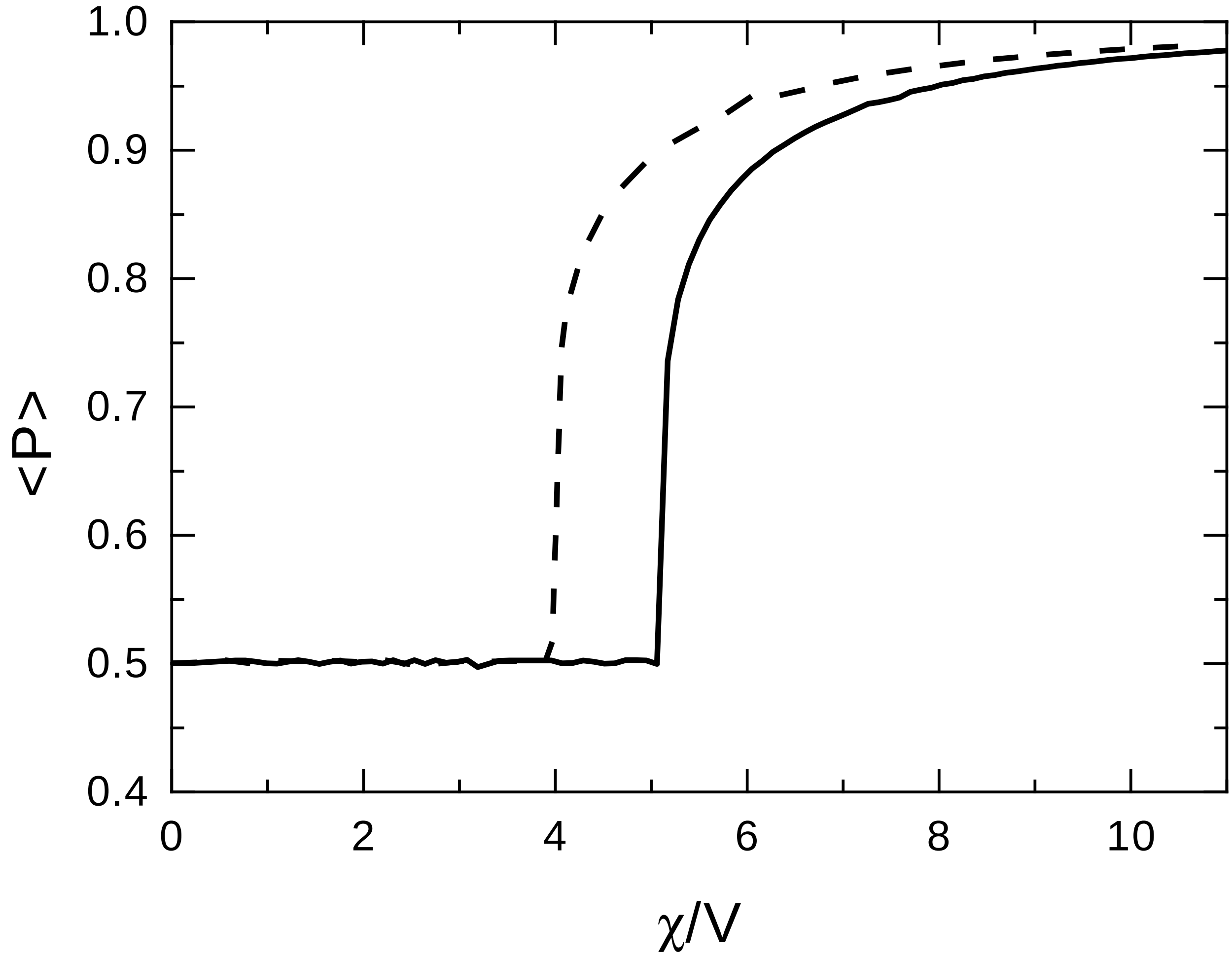
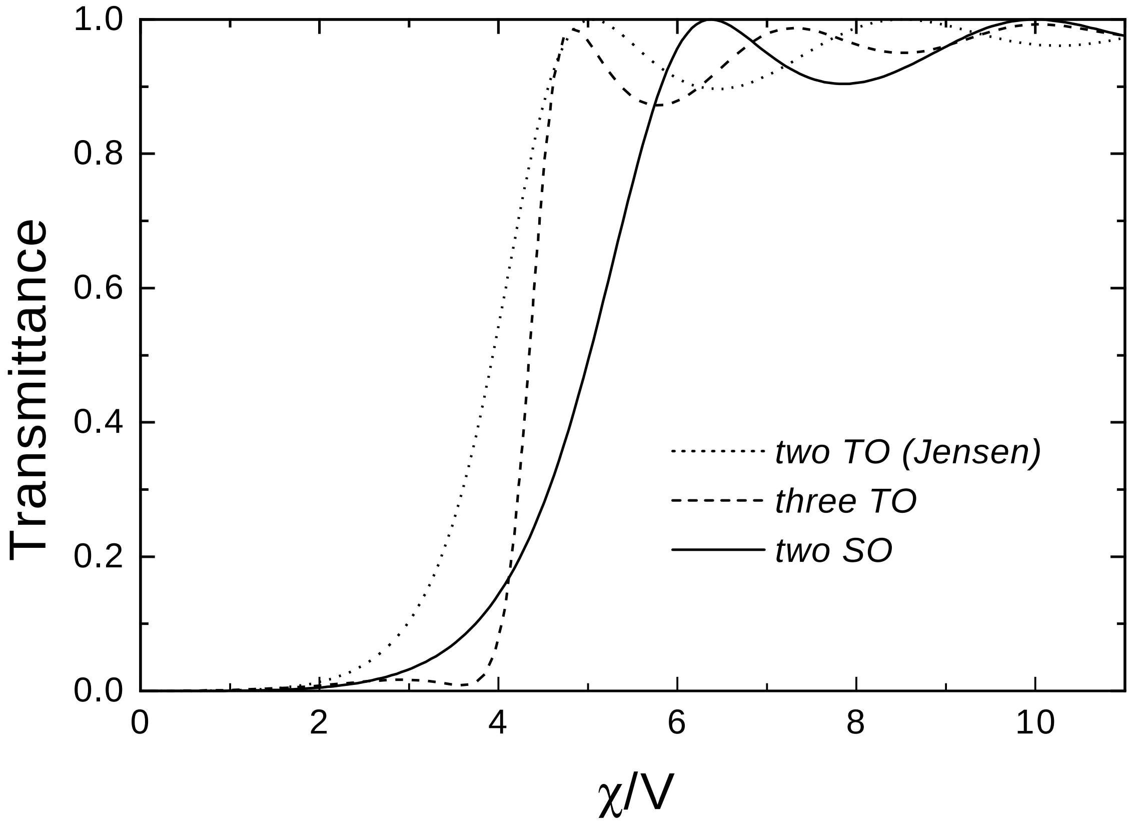


Fig.2



Transmittance
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10
χ/V
two TO (Jensen)
three TO
two SO

Fig.3

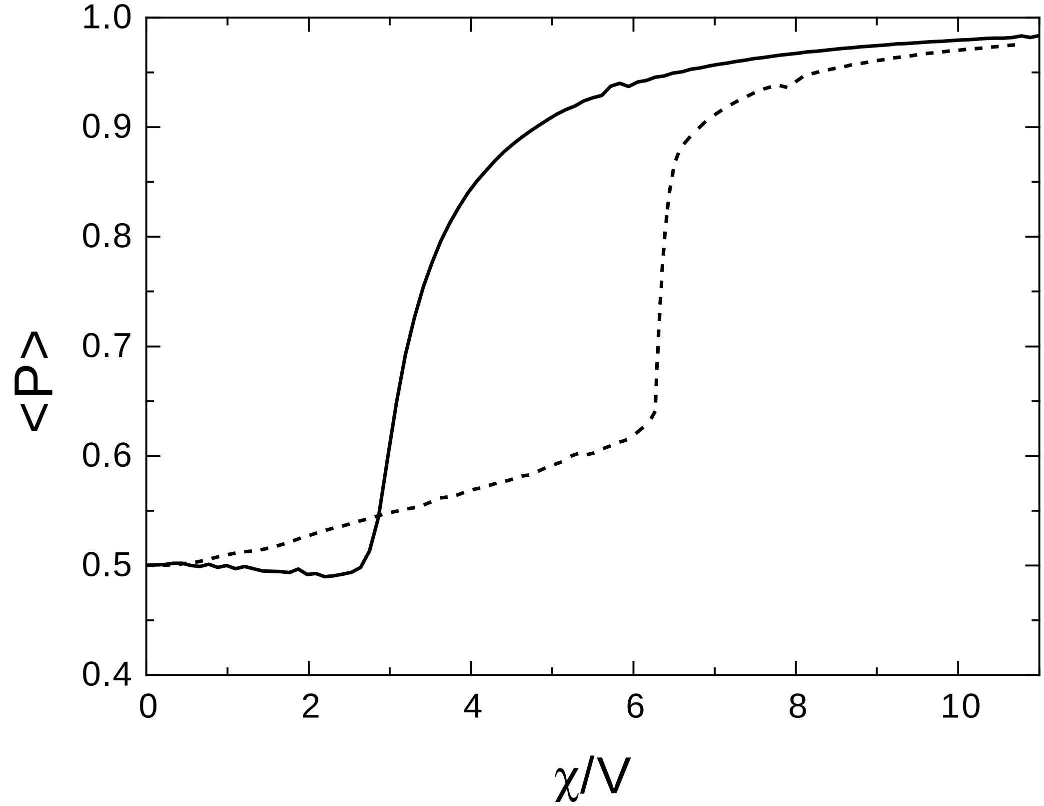


Fig.4

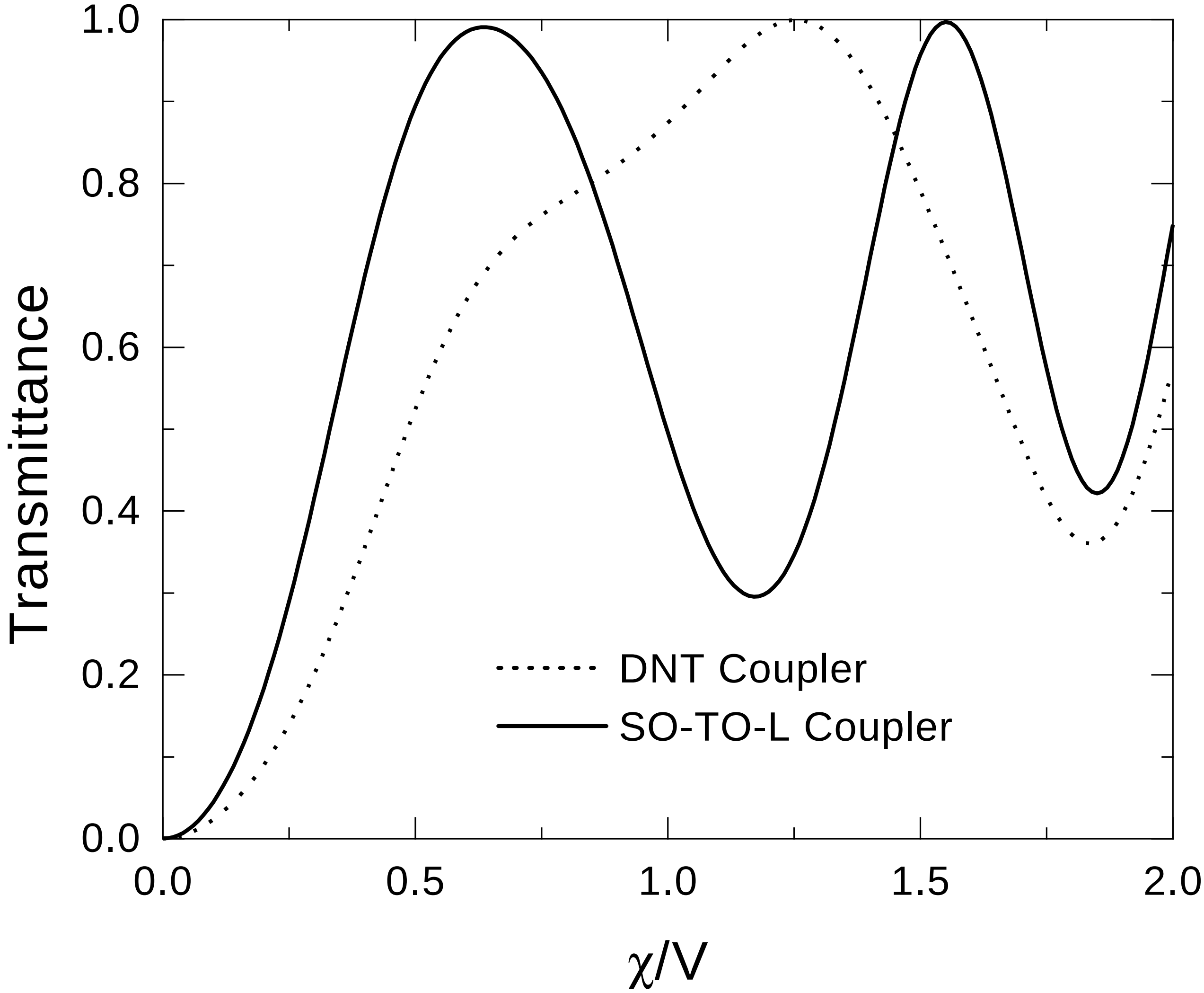

Transmittance
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.5
1.0
1.5
2.0
χ/V
DNT Coupler
SO-TO-L Coupler

Fig.5